AST3.2 Related formulae

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Archaeoastronomically Related Formulae

It is important that findings are supported both culturally and astronomically.

Movement of astral bodies on the celestial sphere is defined mathematically and can be precisely calculated with spherical trigonometry, a fundamental baseline of substantiating archaeoastronomical research.

Spherical Trigonometry

If a spherical triangle is defined as having three angles labeled A, B, and C, and the sides opposite those angles are correspondingly labeled a, b, and c, then the following are the basic formula for solving spherical triangles.

$$\cos a = \cos b \cos c + \sin b \sin c \cos A \quad (1)$$

$$\sin A / \sin a = \sin B / \sin b = \sin C / \sin c \quad (2)$$

$$\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A \quad (3)$$

$$\cos a \cos C = \sin a \cot b - \sin C \cot B \quad (4)$$

These may be used in archaeoastronomy to calculate angles and distance that specify horizon points of celestial bodies and verify field data taken by sighting compass, GPS, and theodolite.

Hour angle of the Sun in degrees =

$$(GMT-12)15 - LONG - (EOT)15 \quad (5)$$

Altitude of the Sun =

$$Arcsin(Sin(LAT)Sin(DEC)+Cos(LAT)Cos(DEC)Cos\ HA)) \quad (6)$$

Azimuth of the Sun =

$$Arcsin\ (Sin(HA)Cos(DEC)/Cos(ALT)) \quad (7)$$

Where HA = Hour Angle, GMT = Universal Time Coordinated, LONG = Longitude, EOT = Equation of Time, LAT = Latitude, DEC = Declination, and ALT = Inclination.

Horizon Deviation

Horizon Deviation occurs when the visible horizon differs from the astronomical horizon

This is common in areas of mountainous terrain and is most pronounced at higher latitudes where the arc of travel for a specific body can yield a significantly different point for rising or setting than that given for the astronomical horizon.

The Sun Above the Tropic of Capricorn, the Equator, and the Tropic of Cancer

When the angular altitude of the horizon at or above level is known by measurement with such as an inclinometer, the latitude of the site in question is taken from a chart or GPS measurement, and the declination of the Sun is derived from the *Nautical Almanac*, the following formula is most useful for calculating the position on the horizon that the Sun will rise.

$$\text{Azimuth of Sun} = \text{Arccos}\,((\text{Sin(DEC)} - \text{sin(LAT)Sin(ALT)})/(\text{Cos(LAT)Cos(ALT)})) \quad (8)$$

Horizon Deviation

Altitude above astronomical horizon	Latitude 13.5°	Latitude 40°
1°	0.22°	1.04°
2°	0.49°	1.99°
4°	1.07°	3.84°
6°	1.69°	5.63°
8°	2.35°	7.37°
10°	3.04°	8.06°

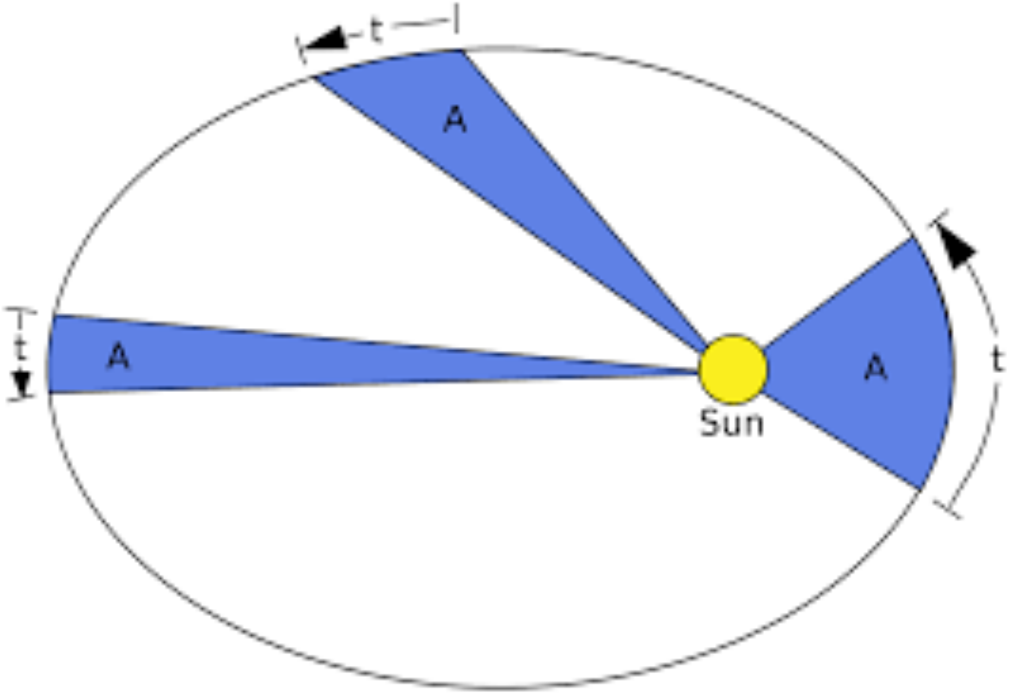
Equation of Time

The difference between apparent solar time and mean solar time is called the *Equation of Time*.

The difference is due to -

The angular velocity of the Earth in its orbit about the Sun varies throughout the year, as described by Kepler's Second Law. At perihelion when the Earth is closest to the Sun, it travels faster than it does at aphelion, so the apparent movement of the Sun increases as well.

The plane of the orbit of the Earth is inclined at 23.439 degrees with respect to the equatorial plane. Therefore, the angular velocity of the relative motion has to be projected from the ecliptic onto the equatorial plane before it can be used to measure time.

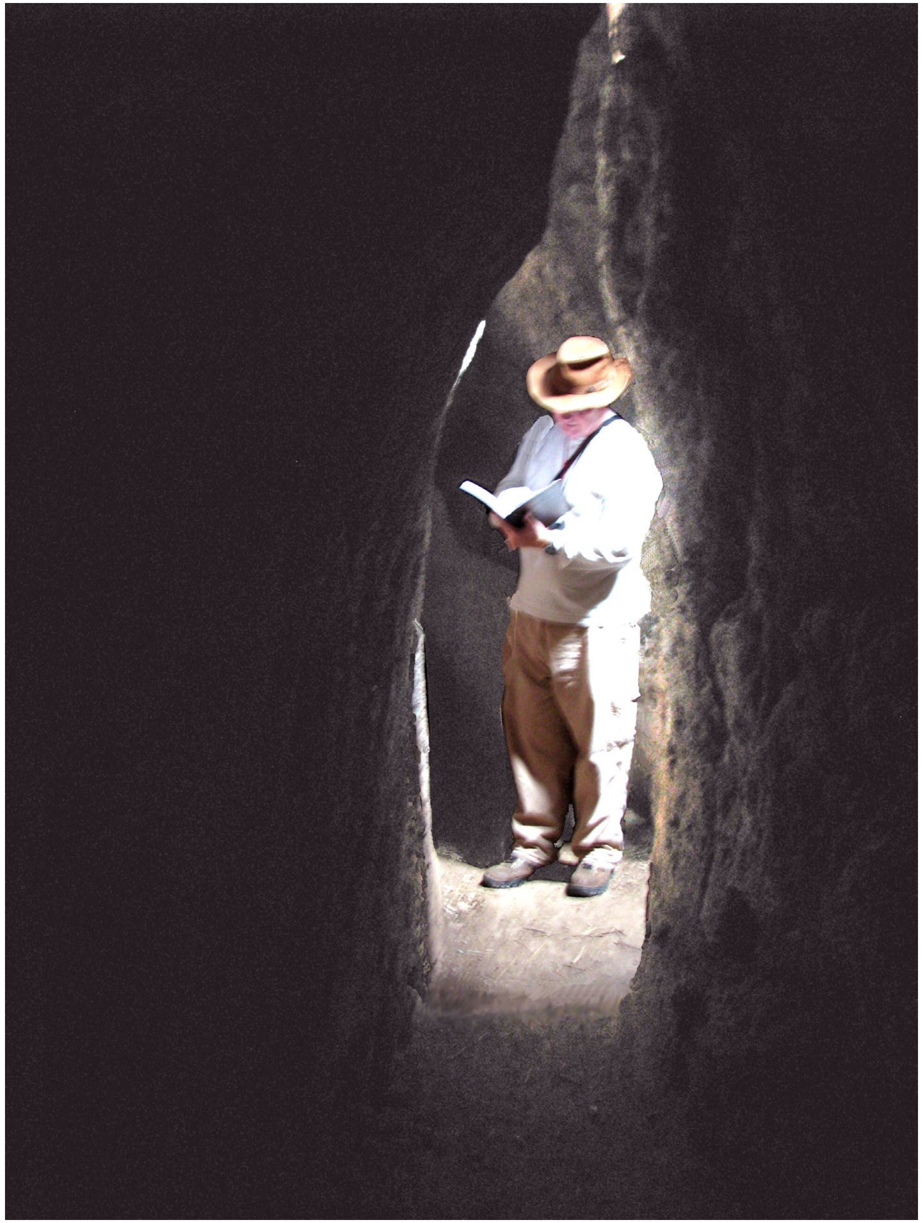


Kepler’s First Law states that all planets move in elliptical orbits about the Sun, and the Sun is positioned at one of the two focal points of the ellipse.

Kepler’s Second Law, the Law of Areas, describes the velocity of planets. The area swept out per time interval (t) remains constant, which gives a faster rate of travel at perihelion and a slower one at aphelion.

Perihelion is when the Earth is closest to the Sun and *aphelion* is when it is furthest away. (Perihelion occurs in January and aphelion is in July.)

Because the main effect of these factors is related to time, this then became known as the *Equation of Time*. The difference is measured in minutes and seconds, corresponding to the amount of time that a sundial would differ when compared with a clock. The Equation of Time can be positive or negative.



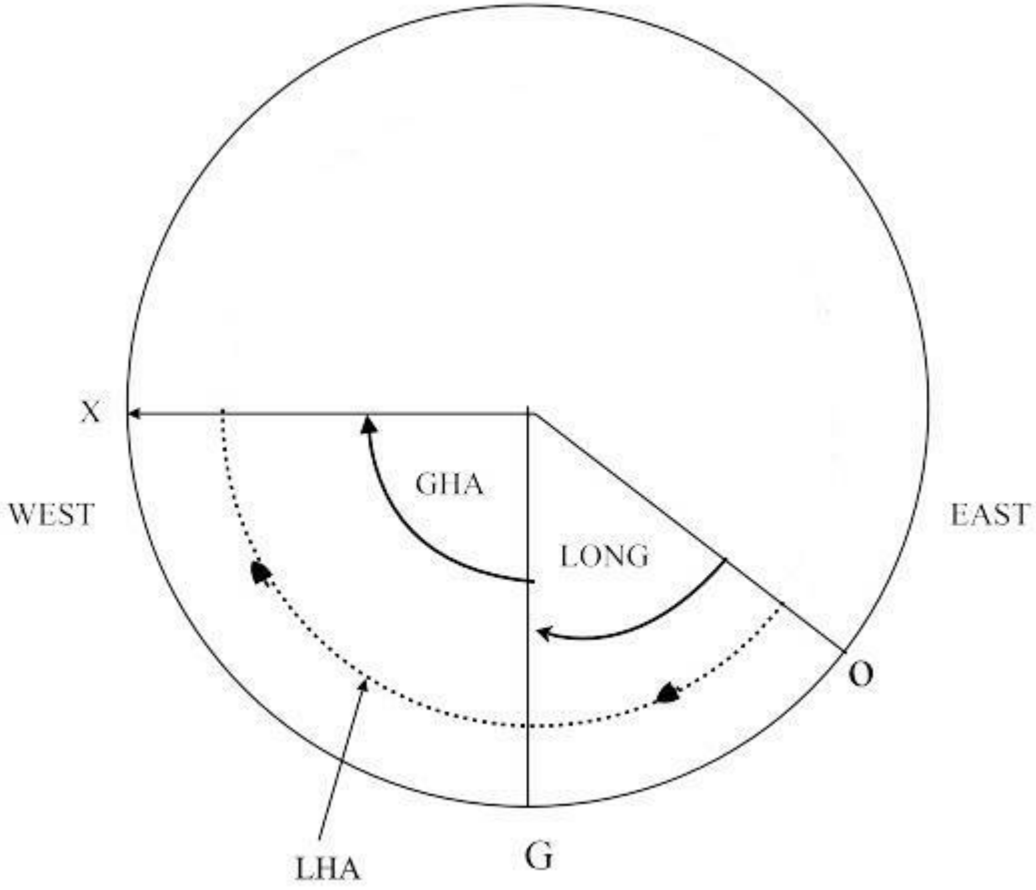
Due to the annual north–south movement of the Sun's apparent position, the variation from the tilt of the Earth's axis, and the varying orbital speed in the Earth’s elliptical orbit, a sundial can be as much as 16 minutes and 33 seconds faster or 14 minutes and 6 seconds slower than the time on a clock. The Earth rotates approximately one degree every four minutes, so a 16-minute displacement corresponds to a shift eastward or westward of about four degrees. A westward shift causes a sundial to be ahead of a clock.

Comparing a sundial with a mechanical clock demonstrates that the solar day is variable in length. Therefore, mean solar time is used instead.

Mean solar time is the average in a solar day. Mean solar time is fixed at a value that remains close to the apparent solar time.

Thus, the differences between apparent solar time and mean solar time are what are described by the Equation of Time

Equation of time = (apparent solar time) - (mean solar time)



O represents the longitude of an observer; G represents the Greenwich meridian; X represents the celestial meridian of a celestial body such as the Sun. LHA is Local Hour Angle.

Equation of time = (apparent solar time) - (mean solar time)

The precise definition is E = GHA of the apparent Sun – GHA of the mean Sun.

Greenwich Hour Angle (GHA) is the hour angle between the Greenwich Meridian and the meridian of a celestial body.

Credits

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